Nigerian Journal of Technology (NIJOTECH)
Vol. 32. No. 2. July 2013, pp. 325 – 332

ISSN: 1115-8443
www.nijotech.com

APPLICATION OF QUEUING THEORY TO PETROL STATIONS IN BENIN-CITY AREA OF EDO STATE, NIGERIA

A. O. Odior
DEPARTMENT OF PRODUCTION ENGINEERING, UNIVERSITY OF BENIN, NIGERIA.
Email: waddnis@yahoo.com

Abstract
The formation of waiting lines is a common phenomenon that occurs whenever the current demand for a service exceed the current capacity to provide that service. Decisions regarding the amount of service to provide must be made frequently in industries even though, it is often impossible to predict accurately when units will arrive to seek service and how much time will be required to provide that service. This paper reports on a study conducted on five petrol stations in Benin City, namely: Oando petrol station Akpakpava, AP petrol station Ugbowo, Total petrol station Iselu, NNPC petrol station Benin-Auchi Road and NNPC Mega filling station Benin-Sapele Road. The average arrival rate of customers per hour for the five petrol filling stations were obtained as 95.4, 97.4, 98.5, 99.6 and 177.8 respectively while the average departure rate of customers per hour were obtained as 86.2, 89, 89.7, 91.7, and 171.6 respectively. The results show that queues exist in each of the five petrol stations. It was also observed that the waiting time in the queue and service time at the five petrol filling stations decrease with increase in the number of servers.

Keywords: Arrival Rate, Customers, Departure Rate, Waiting Lines, Servers

1. Introduction

A common situation that occurs in everyday life is that of queuing or waiting in line. Queues (waiting lines) are usually seen at bus stops, ticket booths, doctor's clinics, bank counters, traffic lights, petrol filling stations, post office and so on. A queuing system consists of one or more servers that provide service of some sort to arriving customers. Customers who arrive to find all servers busy generally join one or more queues (waiting lines) in front of the servers, hence the name queuing systems. Waiting phenomenon is not an experience limited to human beings only: jobs wait to be processed in a machine, planes circles in a stack before given permission to land at an airport, trucks at central market wait for loading and offloading, in warehouse, items wait to be used, incoming calls wait to mature in the telephone exchange [1].

Delays and queuing problems are most common features not only in our daily-life situations such as at a bank or post office, at a ticketing office, in public transportation or in a traffic jam but also in more technical environments, such as in manufacturing, computer networking and telecommunications. They play an essential role for business process re-engineering purposes in administrative tasks. "Queuing models provide the analyst with a powerful tool for designing and evaluating the performance of queuing systems." [2–4]. Whenever customers arrive at a service facility, some of them have to wait before they receive the desired service. It means that the customer has to wait for his/her turn, may be in a line. Customers arrive at a service facility with several queues, each with one server. The customers choose a queue of a server according to some mechanism (e.g., shortest queue or shortest workload) [5, 6]. In some queueing systems customers arrive according to some stochastic process (e.g., a Poisson process) and immediately upon arrival must join one of the queues, thereafter to be served on a first-come first-served basis, with no jockeying or defections allowed. The service times are independent and identically distributed with a known distribution. Moreover, the service times are independent of the arrival process and the customer decisions [7, 8].

Sometimes fuel supply to consumers in Benin City area of Edo state may be epileptic, thereby creating fuel scarcity resulting in long queues of vehicles at filling stations. However, it was observed that even when there was no fuel scarcity, long queues were still prominent in most of the filling stations in Benin City. This became a problem of interest to study so as to know the probable causes of such prominent queues in filing stations even when fuel is available.

1.2 Queueing theory

Queues or queuing theory was first analyzed by A.K Erlang in 1913 in the context of telephone facilities. It is extensively practiced or utilized in industrial setting or retail sector-operations management, and falls under the purview of decision sciences [9]. Queueing theor**y** is the mathematical study of waiting lines, or queues. The theory enables mathematical analysis of several related processes, including arriving at the (back of the) queue, waiting in the queue (essentially a storage process), and being served at the front of the queue. The theory permits the derivation and calculation of several performance measures including the average waiting time in the queue or the system, the expected number waiting or receiving service, and the probability of encountering the system in certain states, such as empty, full, having an available server or having to wait a certain time to be served. Queueing theory has applications in diverse fields including telecommunications, traffic engineering, computing and the design of factories, shops, offices and hospitals [10].

Queue theory has also been found useful in real-world healthcare situations. McClain [11] conducted a research on models for evaluating the impact of bed assignment policies on utilization, waiting time, and the probability of turning away patients. Queue theory was also applied in pharmacy with particular attention to the improvement of customer satisfaction and it was stated that customer satisfaction is improved by predicting and reducing waiting times and adjusting staffing [12]. The theory of queuing as applied in healthcare was presented by Green [13], who discussed the relationship amongst delays, utilization and the number of servers. The use of queuing theory to get approximate results in any queueing system as well as the application of simulation models to refine the obtained results was also studied by Fiems et al. [14]. Ani [15] studied a resource allocation methodology for internet traffic system and explained that the most important quality of service parameters associated with the buffer queuing process at each node (service point) are traffic frame loss rate, traffic frame delay and frame delay variation (jitter). It was also explained that the quality of service parameters relating to specific traffic load and transmission rates are obtained by evaluating the performance of the queuing process at a node for a given buffer size [15].

Queuing theory was employed to address practical questions of sizing and availability assessments, important issues for airspace control systems, indicating useful techniques for the management of critical mission control centers, where many aspects related to human operation, fault tolerance, degraded operation, and demand of service maintenance are basic concerns [16, 17]. A path-combiner was employed in simulation modeling in the study of the effect of data traffic patterns on quality of service parameters order to ensure that the arriving traffic has equal probability of being served in the queue in a random manner. The queue was serviced by a server working at a fixed service rate for all the distributions and using a queue service discipline of first-in-first-out (FIFO) [18].

Application of queuing theory is an attempt to minimize cost through minimization of inefficiencies and delays in a system and there are many problems in health care system which can be solved using queuing theory in operational research. The effectiveness of a queuing model in identifying provider staffing patterns to reduce the fraction of patients who leave without being seen was examined and it was concluded that queuing models can be extremely useful in most effective allocation of staff [9, 19]. The application of queuing theory may be of particular benefit in pharmacies with high volume outpatient workloads and/or those that provide multiple points of service. By better understanding queuing theory, service managers can make decisions that increase the satisfaction of all relevant groups: customers, employees and management [9]. A queueing model was also used by Gorunescu et al. [20] and Siddharthan et al. [21] to determine the main characteristics of the access of patients to hospital, such as mean bed occupancy and the probability that a demand for hospital care is lost because all beds are occupied.

2. Materials and Method

2.1. The queueing system

The data collected were based on the arrival rate and departure rate of cars in the five petrol filling stations studied. This process is also known as the birth and death process and the term birth refers to the arrival of a new customer into the queuing system while the death refers to the departure of a served customer from the petrol filling station. The queues in the petrol filling stations were that of multi-server types {(M/M/s):(FCFS)} where customers arrive according to a Poisson process with infinite source. Where "M" is the arrival process, "M" is the service process and "s" is the number of servers with first come first serve (FCFS) queue discipline. The queueing systems are multiple queueing systems with identical servers in parallel and it is assumed that the arrivals follow a Poisson distribution pattern with arrival rate of "λ" customers per unit of time. It is also assumed that they are served on a first-come, first-served basis by any of the servers. The service time are distributed exponentially, with an average of "μ" customers per unit of time. Queue parameters are found using Little's laws, which states that the long term average number of customers in a stable system is equal to the long term average effective arrival rate multiplied by the average time a customer spend in the system. The utilization factor is the proportion of the system's resources that is used by the traffic which arrives at it. It should be strictly less than one for the system to be functioning effectively.

Data were collected based on the arrival rate and departure rate of cars in the five petrol filling stations studied in Benin-City area of Edo State and they include: Oando Filling Station, Akpakpava with two servers and first in first out (FIFO) queue discipline, AP Filling Station, Ugbowo with three servers, and first in first out (FIFO) queue discipline. Others are, Total Filling Station Uselu, with four servers, and first in first out (FIFO) queue discipline and NNPC Filling Station, Benin-Auchi Road with five servers, and first in first out (FIFO) queue discipline, while data were also collected from NNPC Mega filling Station, Benin-Sapele Road with eight service points, and first in first out (FIFO) queue discipline. The data collected from the five petrol filling stations studied are presented in Table 1.

Table 1. Arrival rates and service rates for the five petrol stations

Period	Petrol Filling Station									
	Oando Station, Akpakpava		AP Station, Ugbowo		Total Station Uselu		NNPC Station Benin-Auchi Road		NNPC Mega Benin-Sapele Road	
	λ (Cars)	μ (Cars)	λ (Cars)	μ (Cars)	λ (Cars)	μ (Cars)	λ (Cars)	μ (Cars)	λ (Cars)	μ (Cars)
8-9am	92	81	103	95	100	96	104	100	165	160
9-10am	94	83	95	88	96	89	98	92	170	162
10-11am	98	90	98	90	99	90	105	95	180	173
11-12pm	87	85	102	95	104	98	91	89	200	190
12-1pm	91	80	97	82	98	85	102	93	160	160
1-2pm	90	84	105	98	95	87	99	90	156	150
2-3pm	102	90	90	87	102	98	101	93	185	179
3-4pm	95	81	94	90	92	80	97	90	174	170
4-5pm	104	95	92	80	99	90	98	95	190	184
5-6pm	101	93	98	85	100	84	100	80	198	188
8-9am	94	83	95	88	96	89	98	92	170	162
9-10am	92	81	98	90	99	90	91	89	174	170
10-11am	98	90	97	82	102	98	101	93	185	179

Period	Petrol Filling Station									
	Oando Station, Akpakpava		AP Station, Ugbowo		Total Station Uselu		NNPC Station Benin-Auchi Road		NNPC Mega Benin-Sapele Road	
	λ (Cars)	μ (Cars)	λ (Cars)	μ (Cars)	λ (Cars)	μ (Cars)	λ (Cars)	μ (Cars)	λ (Cars)	μ (Cars)
11-12pm	87	85	102	95	95	87	97	90	180	173
12-1pm	104	95	105	98	104	98	105	95	200	190
1-2pm	101	93	103	95	100	84	104	100	198	188
2-3pm	91	80	90	87	92	80	100	80	160	160
3-4pm	90	84	94	90	99	90	98	95	165	160
4-5pm	102	90	92	80	100	96	102	93	190	184
5-6pm	95	81	98	85	98	85	99	90	156	150
8-9am	94	83	97	82	96	89	102	93	170	162
9-10am	92	81	98	85	99	90	97	90	185	179
10-11am	95	82	95	88	95	87	91	89	174	170
11-12pm	90	84	92	80	92	80	100	80	198	188
12-1pm	101	92	103	95	102	98	104	100	190	184
1-2pm	102	90	102	95	100	84	98	92	180	174
2-3pm	98	90	98	90	99	90	101	93	156	150
3-4pm	87	85	94	90	98	85	98	95	165	160
4-5pm	91	80	90	87	100	96	99	90	160	160
5-6pm	104	95	105	98	104	98	105	95	200	190
TOTAL	2862	2586	2922	2670	2955	2691	2988	2751	5334	5148

2.2. Litlle's laws

Application of Litlle's laws gives the following relationships among $L_s,\ L_q,\ W_s,\ and\ W_q$ where

Ls = expected number of customers in the system

Lq = expected number of customers in queue

Ws = expected waiting time in system

Wq = expected waiting time in queue

λ = average arrival rate of customers in system

μ = the mean service rate. More generally:

S = number of servers

$$L_s = \lambda W_s \quad (1)$$

$$L_q = \lambda W_q \quad (2)$$

$$W_s = W_q + \frac{1}{\mu} \quad (3)$$

$$\rho = \frac{\lambda}{\mu}$$

More generally,

$$\rho = \frac{\lambda}{\mu * S} \quad (4)$$

where λ is the mean arrival rate, μ is the mean service rate, and "S" is the number of servers. In general, a lower utilization corresponds to less queueing for customers but means that the system is more idle, which may be considered inefficient.

2.3 Two server's case (M/M/2)

The queue system in Oando filling station is a two serve queue system of M/M/2 case and the

queue parameters are obtained as follows:

Average arrival rate of customers, λ = 2862/30 = 95.4/hour (94 customers per hours)

Ls = λWs From equation (1)

$$\therefore\ W_s = \frac{L_s}{\lambda} \quad (5)$$

Expected waiting time in the system, $W_s = \frac{L_s}{\lambda} = \frac{10.0}{95.4} =$ 0.105 hr or 6 min. 18 sec.

Lq = λWq From equation (2)

$$\therefore\ W_q = \frac{L_q}{\lambda} \quad (6)$$

Therefore, expected waiting time in the queue, $W_q = \frac{L_q}{\lambda} = \frac{8.0}{95.4}$ = 0.084 hr. or 5min. 2sec.

The expected service time, μ = expected waiting time in system - expected waiting time in queue

μ = 6min. 18sec. – 5min. 2sec. = 1min. 16sec. (1.26 min.)

Utilization factor $\rho = \frac{\lambda}{s\mu}$ (7)

but λ = 95.4/hour = 1.59/min and s = 2, while μ = 1.26min

$$\therefore\ \rho = \frac{\lambda}{s\mu}\ \frac{1.59}{(2)(1.26)} = 0.63$$

2.4. Three server's case (M/M/3)

The queue system in AP filling station is a three serve queue system of M/M/3 case and the queue parameters are obtained as follows:

Average arrival rate of customers, λ = 2922/30 = 97.4/hour (97 customers per hours)

Expected waiting time in the system, $W_s = \frac{L_s}{\lambda} = \frac{10.0}{97.4} =$ 0.103 hr = 6.18 min. or

Therefore, expected waiting time in the queue, $W_q = \frac{L_q}{\lambda} = \frac{8.0}{97.4}$ = 0.082 hr. = 4.92min. or

Expected service time, μ = 6.18 – 4.92 = 1.26min.

Utilization factor $\rho = \frac{\lambda}{s\mu} = \frac{1.62}{(3)(1.26)}$ = 0.33

2.5. Four server's case (M/M/4)

The queue system in Total filling station is a four serve queue system of M/M/4 case with FIFO and the queue parameters are obtained as follows:

Average arrival rate of customers, λ = 2955/30 = 98.5/hour (99 customers per hours)

Expected waiting time in the system, $W_s = \frac{L_s}{\lambda} = \frac{10.0}{98.5} =$ 0.102 hr = 6.12 min. or

Expected waiting time in the queue, $W_q = \frac{L_q}{\lambda} = \frac{8.0}{98.5}$ = 0.081hr or 4.86min or

Expected service time, μ = 6.12 – 4.86 = 1.26min

Utilization factor $\rho = \frac{\lambda}{s\mu} = \frac{1.64}{(4)(1.26)}$ = 0.33

λ = 98.5/hour = 1.64min

Þ= λ /sμ = 1.64/4 x 1.26= 0.33min

2.6. Five server's case (M/M/5)

The queue system in NNPC filling station is a five serve queue system of M/M/5 case with FIFO and the queue parameters are obtained as follows:

Average arrival rate of customers, λ = 2988/30 = 99.6 /hour or 100 customers per hour

Expected waiting time in system, $W_s = \frac{L_s}{\lambda} = \frac{10.0}{99.6} =$ 0.10 hr. = 6 min.

Expected waiting time in queue, $W_q = \frac{L_q}{\lambda} = \frac{8.0}{99.6}$ = 0.082 hr. = 4.8 min.

Expected service time, μ = expected waiting time in system - expected waiting time in queue

$\therefore \mu =$ 6 – 4.8 = 1.2min

Utilization factor $\rho = \frac{\lambda}{s\mu} = \frac{1.66}{(5)(1.2)}$ = 0.28

2.7. Eight server's case (M/M/8)

The queue system in NNPC Mega filling station is an eight serve queue system of M/M/8 case with FIFO and the queue parameters are obtained as follows:

Average arrival rate of customers, λ = 5334/30 = 177.8 /hour or 178 customers per hour

Expected waiting time in system, $W_s = \frac{L_s}{\lambda} = \frac{10.0}{177.8} =$ 0.056 hr. = 3.36 min.

Expected waiting time in queue, $W_q = \frac{L_q}{\lambda} = \frac{8.0}{177.8}$ = 0.045 hr. = 2.7 min.

Expected service time, μ = 3.36 – 2.7 = 0.66min

Utilization factor factor $\rho = \frac{\lambda}{s\mu} = \frac{2.96}{(8)(0.66)}$ = 0.56

3. Results and Discussion

The computed results for expected waiting time in the system, the expected waiting time in queue, the average arrival rate of customers in the system and the mean service rate are presented in Table 2. The utilization factor was also computed for each of the filling stations and the results showed that it was less than one for each of the stations. The results also revealed that there was a rush of customers into the system during the early hours of the day, when people are rushing to their various offices; during lunch-break; at the end of the day's activities when people are retiring to their various homes; during the weekends as well as during scarcity periods. The queue situation on Fridays and weekends in the patrol stations was always different from other weekdays. It was observed that motorists formed long queues at the various stations waiting to be served. Also the arrival rate was always found to be higher than the service rate for each of the stations during the weekends

Table 2. Summary of results

S/N	Petrol Filling Station	No of Servers	Average arrival/hr	Average departure/hr	L_S	L_q	W_S	W_q	μ	ρ
1	Oando filling station	2	95.4	86.2	10	8	6.30	5.04	1.26	0.63
2	AP filling station	3	97.4	89	10	8	6.18	4.92	1.26	0.33
3	Total filling station	4	98.5	89.7	10	8	6.12	4.86	1.26	0.33
4	NNPC filling station	5	99.6	91.7	10	8	6.00	4.80	1.20	0.28
5	NNPC MEGA filling station	8	177.8	171.6	10	8	3.36	2.70	0.66	0.56

The results revealed that the average arrival rate of customers to the filling stations was always higher than the average departure rate of customers from the filling stations and this explained while queues always existed in all the filling stations. This is clearly seen in the graph of arrival rate and service rate for Oando and NNPC petrol stations in Figure 1 which shows that the arrival rate was always higher than the service rate. Figure 2 also shows that the rate at which customers entered any of the stations for service was always higher than the rate at which customers leave the station after being served. This is the reason while there was always queue in any of the stations studied. However, the NNPC Mega filling station with eight (8) service points has almost the same arrival rate and service rate of customers. This is closely followed by the NNPC filling station with five (5) service points.

4. Conclusion

Queueing systems are more prevalent in *our* increasingly congested and urbanised *society*. The queue might involve data waiting for processing, equipment parts waiting in an assembly line or people waiting in line at various types of business centers. Queue theory is an important tool used to model many supply chain problems and it is used to study situations in which customers form a line and wait to be served by service or manufacturing facility. The results from the study revealed that utilization factor for each of the filling stations was less than one (1) which showed that the servers were duly attending to their customers. The utilization factors were obtained as 0.63, 0.33, 0.33, 0.28 and 0.56 for Oando filling station, AP Filling Station, Total filling station, NNPC filling station and NNPC Mega filling Station. It was observed that only the NNPC Mega filling station was using all its service points. It was therefore suggested to the filling stations to always endeavour to use all their service points (pumps) at peak periods to reduce queues at such stations

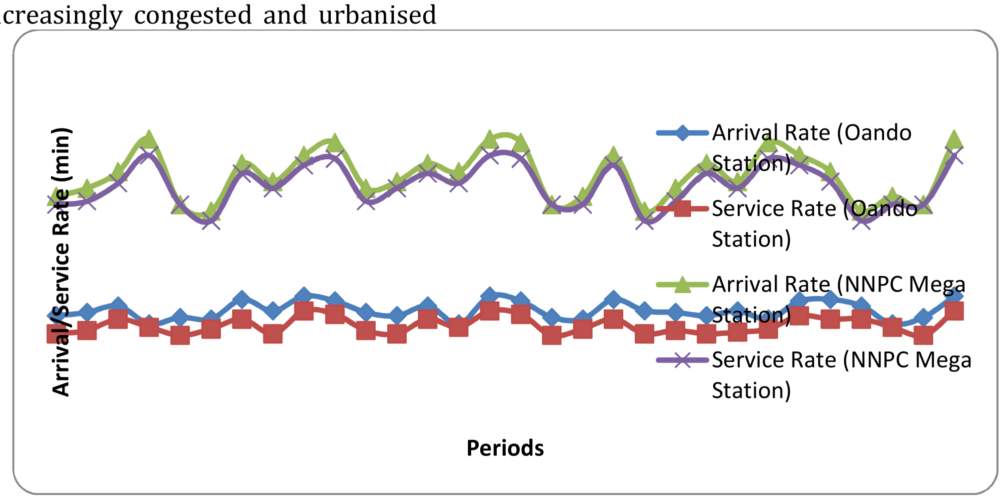

Fig.1. Arrival rate and service rate for two petrol stations

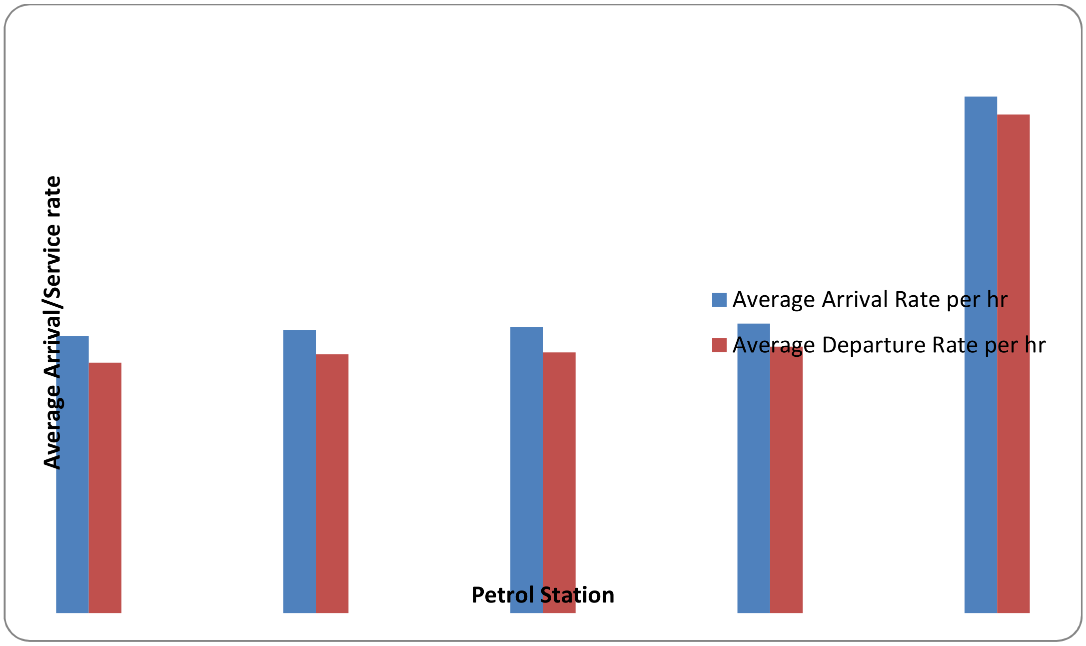

Fig. 2: Average arrival and departure rate for the five petrol stations

References

[1] Omid A., Shahram A., Alireza A., Ali H. and Safar M. Cellular Manufacturing System Design with Queueing Approach, *Journal of Basic and Applied Scientific Research*, Vol. 2, No.11, 2012, pp 11884 - 11890

[2] Banks, J., Carson, J. S., Nelson, B. L., Nicol, D. M. *Discrete-Event System Simulation*, 3rd edition. Prentice Hall, Upper Saddle River, New Jersey, 2001.

[3] Cohen, J. W. Some Aspects of Queueing Theory, *Journal of Probability and Mathematical Statistics*, Vol. 28, No. 2, 2008, pp 55–67

[4] Abolnikov, L., Dshalalow, J. E., Dukhovny, A. M. On Some Queue Length Controlled Stochastic Processes,"*Journal of Applied Mathematics and Stochastic Analysis,* Vol. 3, No. 4, 1990, pp 227 – 244

[5]. Deb, R., Rajat, K., Optimal control of bulk queues with compound Poisson arrivals and batch service, *Journal of Operations Research*, Vol. 21, No.4, 1984, pp 227-245.

[6] Adan, I.J.B.F., Boxma1, O.J., Resing, J.A.C. "Queuing models with multiple waiting lines," Department of Mathematics and Computer Science, Eindhoven University of Technology, Netherlands, 2000.

[7] Whitt, W. Deciding Which Queue to Join: Some Counterexamples, *Journal of the Operational Research Society*, Vol. 34, No. 1, 1986, pp. 55-62

[8] Bailey,N.T. A Continuous Time Treatment of Simple Queue Using Generating Functions", *Journal of the Royal Statistical Society*, Vol.16, 1954, pp 288-291.

[9] Mehandiratta, R. Applications of Queuing Theory in Health Care, *International Journal of Computing and Business Research*, Vol. 2, No. 2, 2011, pp 36 – 48

[10] Gudmundsson, D. and Goldberg, K. Optimizing robotic part feeder throughput with queueing theory", *Assembly Automation*, Vol. 27, No. 2, 2007, pp.134 – 140

[11] McClain, J.O. Bed planning using queuing theory models of hospital occupancy: a sensitivity analysis, *Inquiry*, Vol.13, 1976, pp 167-176.

[12] Nosek, Jr., R.A. and Wilson, J.P. Queuing theory and customer satisfaction: a review of terminology, trends, and applications to pharmacy practice. *Hospital Pharmacy Journal*, Vol. 36, 2001, pp 275-279.

[13] Green, L.V. Queueing analysis in healthcare, in Patient Flow: Reducing Delay in Healthcare Delivery, Hall, R.W., ed., Springer, New York, 2006.

[14] Fiems, D., Koole, G. and Nain, P. (2007) Waiting times of scheduled patients in the presence of emergency requests. http://www.math.vu.nl/~koole/articles/report05a/art.pdf**.** Accessed on August 6, 2007.

[15] Ani, C. I. Resource Allocation Methodology for Internet Heterogeneous Traffic, Nigerian Journal of Technology, Vol. 23, No. 1, 2004, pp 16 - 21

[16] Pizzo, W. N. and Cugnasca, P. S. Application of Queuing Theory for Availability Assessment in Airspace Control Systems, *Journal of the Brazilian Air Transportation Research Society*, Vol. 3, No. 1, 2007, pp 53-65

[17] Jackson, J.R. Jobshop-Like Queueing Systems, *Journal of Management Science*, Vol. 50, No. 12, 2004, pp. 1796-1802.

[18] Ani, C.I. and Isimijola, S.O. The Effect of Data Traffic Patterns on QoS Parameters, Nigerian Journal of Technology, Vol. 28, No. 2, 2009, pp 23 - 28

[19] Green, L.V., Soares, J., Giglio, J.F. & Green, R.A.Using queuing theory to increase the effectiveness of emergency department provider staffing", *Academic Emergency Medicine Journal*, Volume 13, No. 1, 2006, pp 61-68.

[20] Gorunescu F., McClean, S.I. and Millard, P.H. A Queueing Model for Bed-Occupancy Management and Planning of Hospitals, *Journal of the Operational Research Society*, Vol. 53, 2002, pp 19–24

[21] Siddharthan, K., Jones, W.J. and Johnson, J.A. A priority queuing model to reduce waiting times in emergency care", *International Journal of Health Care Quality Assurance*, Vol. 9, No. 5, 1996, pp.10 – 16