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BEYOND SADDLE-NODE BIFURCATION POINT OF ILL-CONDITIONED POWER SYSTEMS: A CASE STUDY OF NIGERIAN NATIONAL GRID

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Abstract
Estimating the margin for voltage collapse is a critical concern for power system utility companies and stakeholders in developing power systems. A common approach uses Euclidean distance in continuation power flow to address singularity-induced network constraints. However, this method fails to capture exact topological properties, leading to overly conservative stability margin predictions. To overcome these limitations, this study introduces the arc-length path to the existing model, leveraging numerical simulations with varying step sizes. The optimal step size value was determined through various simulations, including 19 incremental step size adjustments and 8-step size reductions. The results of these simulations validated the findings. Notably, combining arc length and Euclidean distance for margin estimation reveals that the latus rectum, corresponding to local parameterization, and the margin to voltage collapse are equidistant from the saddle-node bifurcation point.

1.0 INTRODUCTION
The Saddle Node Bifurcation (SNB) or Fold Point (FP) is the location where the equilibrium of two dynamic systems coalesces as the parameter changes [1]. On the other hand, an ill-conditioned power system is characterized by frequent voltage collapse occasioned by insufficient reactive reserve, low voltage profiles, high reactive power demand, and limited power transfer capabilities among other factors [2]. Recently, the estimation of the load distance between the current operating point and the knee point of the power system has been a serious concern for [2], [3], and [4]. Various approaches such as [5], [6], and [7] are some of the techniques for tracking the load distance and the point of collapse of the studied system. However, multiple power flow calculations, large coefficient matrix concerns, and eigenvector initialization are the drawbacks of those approaches [8].

The step increase of real power at a particular voltage-controlled bus and the observation of the corresponding variation of voltages at a load bus is the idea adopted by [9], [10], and [11]. Onah and Jagun [12], adopted the idea that the maximum loadability limit can be evaluated by solving the nonlinear nature of PV-curve. Explicitly put, if the real power voltage curve of a bus is built by solving the nonlinear equation [12], an assumption that the quadratic

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approach assumes a second-order approximation for the plot of $V_a = X(\lambda)$, does not hold at all times. The approach investigated by Jagun and Onah [12], is based on equations (1) and (2).

$$P_a = P_{a0}(1 + \lambda f) = \sum_b^N V_a V_b Y_{ab} \cos(\delta_a - \delta_b - \theta_{ab}) \quad (1)$$

$$Q_a = Q_{a0}(1 + \lambda g) = \sum_b^N V_a V_b Y_{ab} \sin(\delta_a - \delta_b - \theta_{ab}) \quad (2)$$

Recently, Onah et al [13], obtained the point of collapse using a voltage collapse proximity indicator. Although the result was accurate the approach is rigorous owing to the continuous variations in the load.

Moreover, it is possible to estimate the margin to voltage collapse using the P-V curve but the approach is computationally intensive owing to the repetitive load flow studies. Again, at the voltage collapse point of the P-V curve, the Jacobian matrix of the power flow equation is singular. To get over the constraints, Onah et al [14] introduced load parameters to power flow equations in (1) and (2) to obtain an accurate voltage-load parameter curve in equations (3) and (4) as follows.

$$P_a = P_{a0} + \text{Ɯ} P_{a0} \quad (3)$$
$$Q_a = Q_{a0} + \text{Ɯ} Q_{a0} \quad (4)$$

Where, Ɯ is the load parameter, the approach uses the predictor-corrector scheme to extrapolate the curve and obtain an exact value of the collapse point, current operating point, and the margin to voltage collapse. However, the scope did not exceed the limit of local bifurcation theory. Reference [15] explored the shortest path to the boundary of the saddle-node bifurcation point, comparing arc length and Euclidean distance. This study highlighted the importance of steering operating points away from critical power system thresholds, pinpointing a valuable research direction for future investigators.

The focus of this research is to obtain an accurate margin to voltage collapse by optimal selection of the step size for predictor-corrector processes. This is because the implicit assumption that the step size has to be relatively small for quicker convergence of the corrector does not hold in all conditions. Therefore, attempts were made to obtain an accurate value of step size for the predictor-corrector scheme.

On the other hand, the equidistance of the latus rectum and the directrix of the saddle-node bifurcation point of the loading parameter-voltage characteristics of the Nigerian power system give a simple and easier way of getting to know the effects of overloading the energy source. The chosen step size is so infinitesimal to enable quicker convergence of the corrector from the predicted solution. The following assumptions were made for the proposed work:

i. The Jacobian matrix of the 50 bus-Nigerian National Grid (NNG) is singular at the saddle-node bifurcation point.
ii. Normal vector at SNB cannot be obtained using the conventional load flow method.
iii. The voltage-loading parameter characteristics of the 50 bus-NNG is convex
iv. The parabolic equation is given by $V^2 = -4a\lambda$.
v. The influence of a generator's reactive power limit is not considered

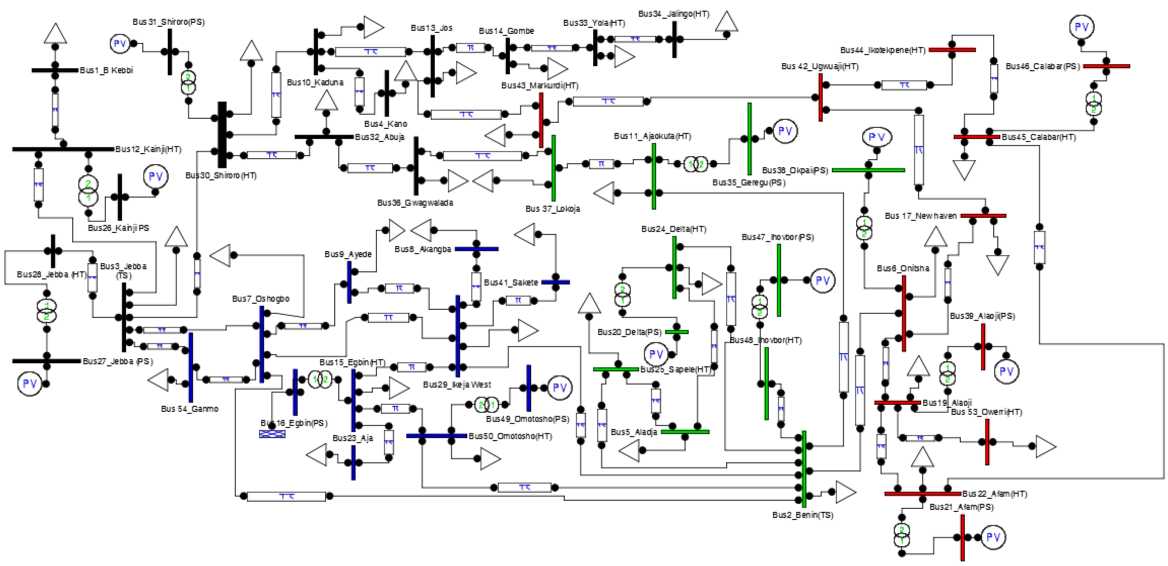

Figure 1: One-Line Diagram of 50-Bus Nigerian national grid

2.0 MATERIALS AND METHOD

The study adopted a research and development design. The network comprises a 50-bus Nigeria National Grid (NNG) as shown in Figure 1. Power System Analysis Toolbox (PSAT) is employed for the analysis. An algorithm for voltage collapse prediction using a hybrid continuation power flow path tracing technique (CPF-PTT) was developed. The use of the CPF technique for the prediction of voltage collapse was investigated.

2.1 Algorithm for the CPF-PTT

i. Start
ii. Investigate the best load flow for the study
iii. Run the Newton Raphson's load flow
iv. Is the Jacobian matrix singular at the fold point?
v. If no go to step ii
vi. If yes, introduce load parameter Ɯ at equations (1) and (2) and go to step vi
vii. Take a small-sized step in a direction tangent to the solution path
viii. Obtain the tangencies of the two solution paths of Figure 3
ix. Determine the point of convergence.
x. Stop.

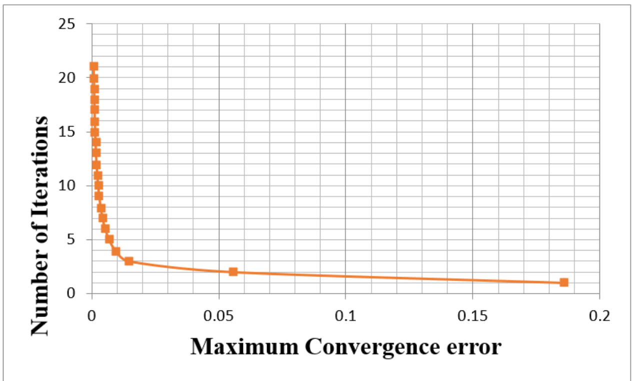

Figure 2: Iwamotor load flow solver

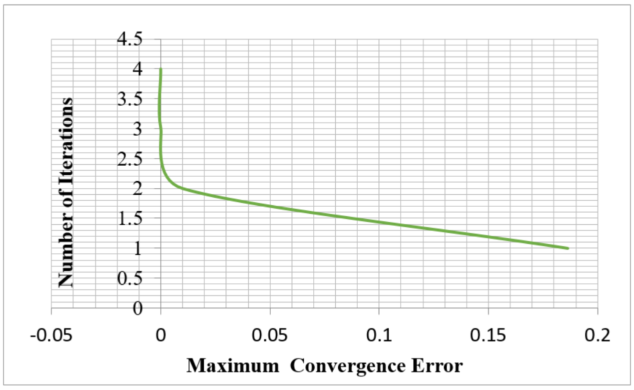

Figure 3: Newton Raphson load flow solver

3.0 RESULTS AND DISCUSSION

3.1 Different Power Flow Solvers of Base Case Power Flow Solutions

To run the continuation power flow studies, an attempt was made on the Iwamotor solver. The simulation converged on the 21st iteration at 0.00 mismatches as seen in Figure 2. Because the time per iteration was considerably large, another attempt was made on Newton Raphson's solver and the convergence is at 4 seconds. Special consideration was given to Newton Raphson on the account of time per iteration and ease of convergence as can be observed in Figure 3.

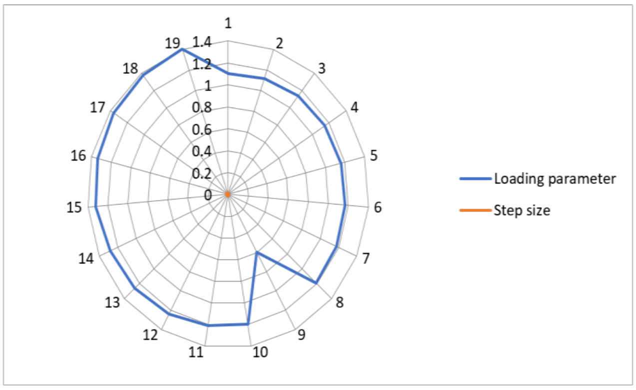

Figure 4: The variation of step size with the loading parameter at expanded range

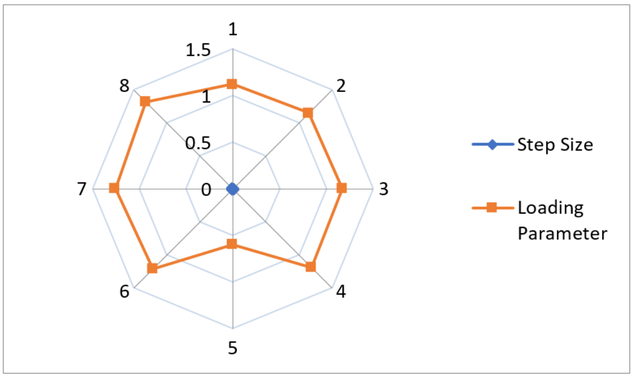

Figure 5: The variation of step size with the loading parameter at a reduced range

3.2 Optimal Selection of Step Size for CPF

Since the selection of step size for the CPF is a serious concern, it becomes expedient to obtain an optimal value of the step size for an accurate value of the loadability limit. It is observed that over-selection and under-selection of step sizes do not give accurate results. To get over these constraints, two different approaches were adopted. First, the ranges of step sizes in expanded form for various values of SNB were obtained. Secondly, the ranges were reduced to validate the result in the first step as shown in Figure 4. In a bid to obtain an accurate SNB and an accurate margin to voltage collapse, the CPF simulations were carried out in 19 different step size increments, the step size was selected from a range of 0.001 to 0.02. The 9th step corresponds to a step size of 0.6 as espoused in Figure 4.

Similarly, the reduction of the range of the step sizes from 0.001 to 0.02 and from 0.02 to 0.016 at 8 different simulations corroborated and validated the result when the step size is 0.6 as shown in Figure 5.

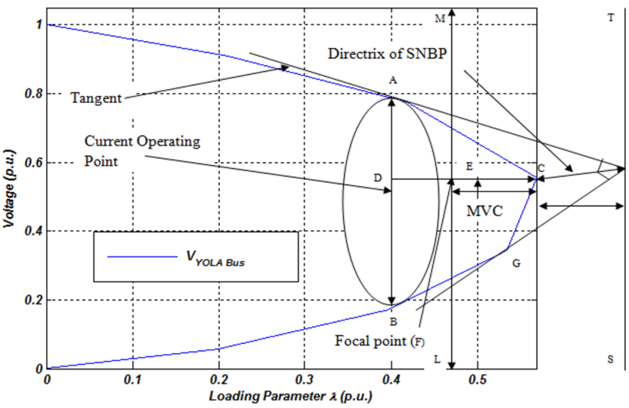

Figure 6: The loading parameter-voltage curve

Figure 6 contains a set of points in a plane that has an equal distance away from a given point and a given line. The point is the focus of the loading parameter-Voltage curve. The line is known as the directrix which is perpendicular to the axis of the curve. The perpendicularity shows that the line can never touch the curve. From Figure 6, /EC/ = /CF/. /EC/ is the distance between the focus of the curve and the fold point of the dynamic system. Meanwhile, /CF/ is the directrix of the curve which is the distance from the saddle-node bifurcation point to the annihilation point of the predictors of the stable and unstable paths of the power system. The points of tangencies to the loading parameter-Voltage curve of the two dynamic systems yield the current operating point of the power system. The distance between the current operating point /AB/ and saddle-node bifurcation point (C) gives the margin to the voltage of the Yola bus of the NNG. The distance of the latus rectum /LM/ from the SNBP is equal to the distance of the directrix /ST/ from the SNBP. /LM/ and /ST/ are two parallel lines that are equidistant from the SNBP of the power system. It should be noted that the /ST/ is the physical parameterization (PP) while the /FC/ is the local parameterization (LP). Yola-Bus is taken as a reference bus for the analysis because it has the lowest voltage on the 50-bus Nigeria National Grid after load flow studies. That shows that at the latus rectum, it is possible to obtain a possible picture of the effects of the voltage collapse on the synchronous generator when two dynamic systems coalesce beyond SNBP. The equation of the parabolic power system is given as $V^2 = -4a\lambda$ as observed by [16]. Where V stands for y as can be seen in Figure 7. The concept of blow-up time of the upper and lower region of a similar curve can be investigated as emphasized in [17]. However, it is outside the scope of this study and can be analyzed using the approach adopted in [18].

From Figure 8, the current operating point is seen as 0.4p.u and the fold point of the ill-conditioned power system is 0.58p.u. The margin to voltage collapse is seen as 0.18p.u. The upper region of the curve is a stable region that is obtainable in practice and the lower region is the unstable region that is not obtainable in practice. Again, from Figure 6, $/AC/^2 = /AD/^2 + /DC/^2$, /AC/ = 0.28p.u. The margin to the voltage collapse is 0.18p.u. It is important to state that it would have been difficult to have stable and unstable regions of interest if the Yola bus exhibited a high voltage profile. This factor makes the Nigerian National Grid an ill-conditioned power system as elucidated in [2].

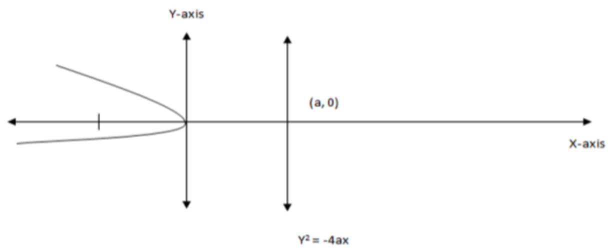

Figure 7: Parabolic power system

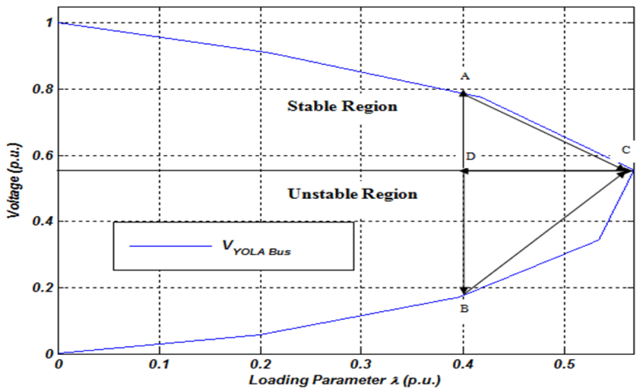

Figure 8: Loading parameter-Voltage characteristics

3.3 Contributions to Knowledge

The following contributions to knowledge were made:

i. Obtaining the optimal step size for the predictor-corrector scheme
ii. Obtaining the latus-rectum of the voltage-loading parameter characteristic of the 50-bus Nigerian National Grid as the local parameterization (LP)
iii. Obtaining the directrix of the saddle-node bifurcation as the physical parameterization (PP)

iv. Establishment of the fact that the distance between the current operating point and the saddle-node bifurcation point is equal to the point of collapse and the physical parameterization.
v. Establishment of the fact that the latus which is the directrix of the lamda – voltage characteristics of the power system can help to determine what happens to the synchronous generators beyond saddle-node bifurcation point

4.0 CONCLUSION

The step size of the continuation power flow should be chosen taking into consideration the convergence of solutions and the computation speed. The choice of selection of appropriate continuation parameters is relevant to the convergence of solutions. Researchers are hard put to it selecting a continuation parameter for optimal convergence of solutions. The study identified an optimal, efficient, and easier way to select the continuation parameter for fast and accurate results. Going further, the study established the possibility of extrapolating the loading parameter voltage characteristics beyond the fold point. The study proved that the margin to voltage collapse is equal to the latus rectum of the parabolic ill-conditioned power system.

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